Overlap-and-add for fast linear filtering

FIR filters

Fast convolution with streaming input

Herman Kamper

Linear filtering using the FFT

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n] \qquad y[n] = x[n] * h[n]$$

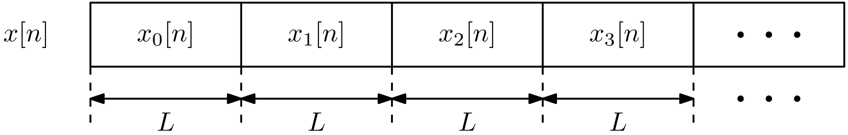
Calculate the discrete convolution using the FFT:

- Length of $x[n]$ is M
- Length of $h[n]$ is P (FIR)
- Zero pad $x[n]$ and $h[n]$ to length $N \geq M + P - 1$
- $y[n] = \mathrm{IFFT}\left\{X_{\mathrm{zp}}[k] \cdot H_{\mathrm{zp}}[k]\right\}$

Zero padding

But $x[n]$ is often streamed in, i.e. not bounded in length.

Overlap-and-add



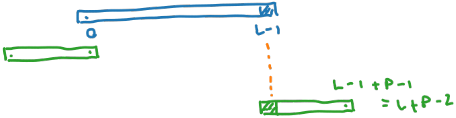
$$x[n] = \sum_{i=0}^{\infty} x_i[n] \quad \text{where} \quad x_i[n] = \begin{cases} x[n] & \text{if } n \text{ is window } i \\ 0 & \text{otherwise} \end{cases}$$

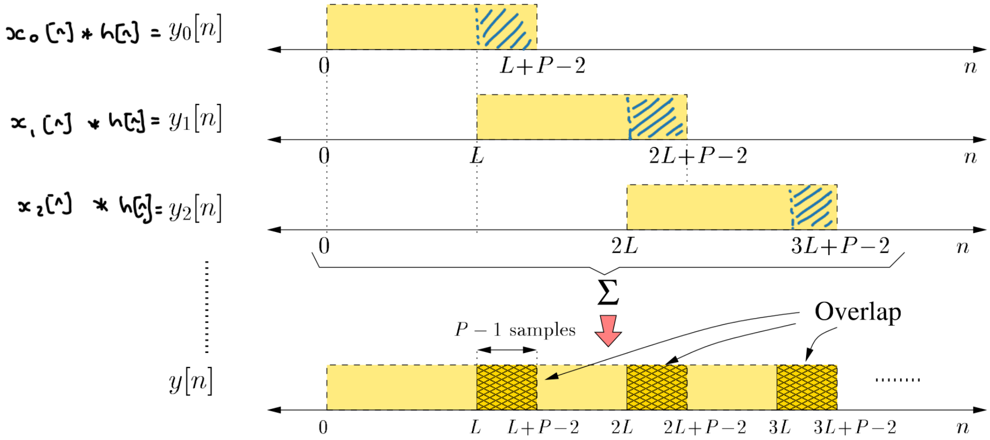
$$\begin{aligned} y[n] = h[n] * x[n] &= h[n] * \left(\sum_{i=0}^{\infty} x_i[n] \right) \\ &= \sum_{i=0}^{\infty} h[n] * x_i[n] \\ &= \sum_{i=0}^{\infty} y_i[n] \quad \text{where} \quad y_i[n] = h[n] * x_i[n] \end{aligned}$$

$$y_0[n] = \overset{P}{h[n]} * \overset{L}{x_0[n]}$$

$n = 0:$

$n = ?:$

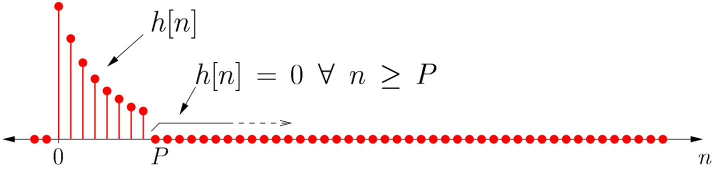


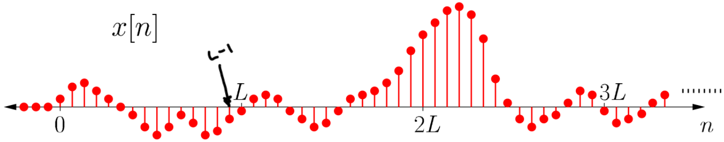


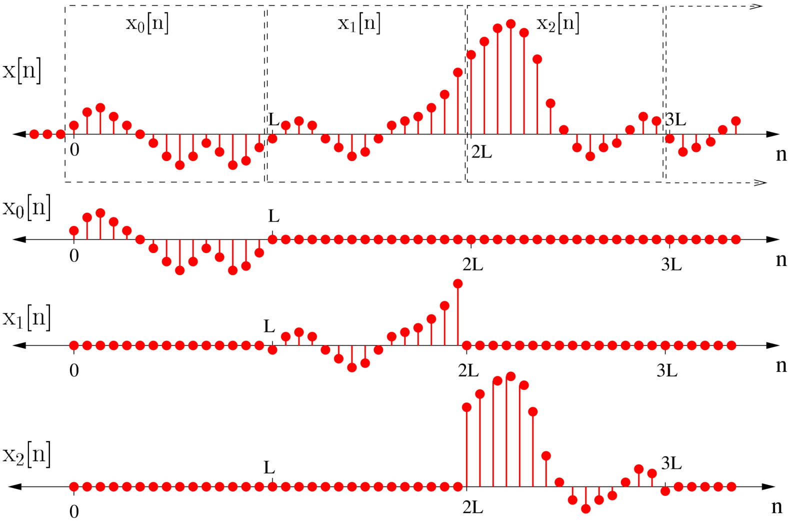
$x_0[n] * h[n] = y_0[n]$
0
$L+P-2$
n
$x_1[n] * h[n] = y_1[n]$
0
L
$2L+P-2$
n
$x_2[n] * h[n] = y_2[n]$
0
$2L$
$3L+P-2$
n
Σ
$P-1$ samples
Overlap
$y[n]$
0
L
$L+P-2$
$2L$
$2L+P-2$
$3L$
$3L+P-2$
n

Overlap-and-add example

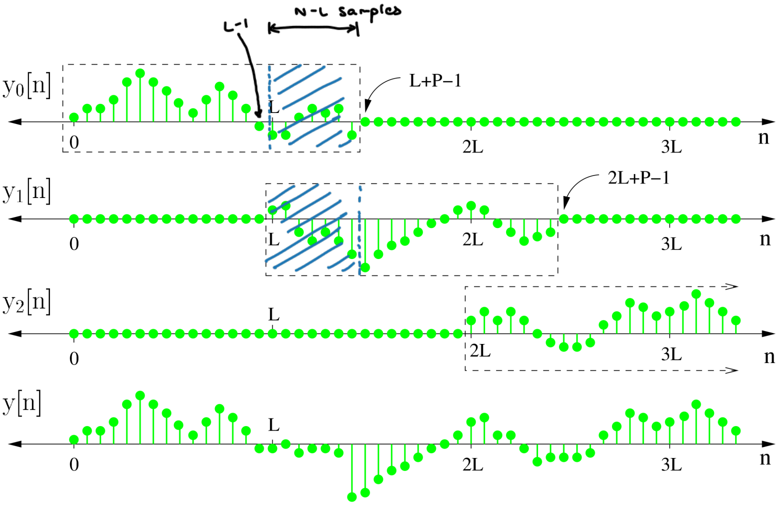
FIR







x[n]
$x_0[n]$
$x_1[n]$
$x_2[n]$
0
L
2L
3L
n
$x_0[n]$
0
L
2L
3L
n
$x_1[n]$
0
L
2L
3L
n
$x_2[n]$
0
L
2L
3L
n



N-L samples
L-1
$y_0[n]$
L
L+P−1
0
2L
3L
n
$y_1[n]$
2L+P−1
0
L
2L
3L
n
$y_2[n]$
L
0
2L
3L
n
$y[n]$
L
0
2L
3L
n

Overlap-and-add procedure

- Choose a suitable block length L
- Zero pad $h[n]$ to length $N \geq L + P - 1$
- Calculate $\underset{\text{zp}}{H}[k] = \text{FFT}\{\underset{\text{zp}}{h}[n]\}$
- For each L-sample block of the input sequence:
  - Zero pad to length N
  - Calculate the FFT
  - Multiply with $\underset{\text{zp}}{H}[k]$
  - Calculate the IFFT
  - Add to $y[n]$, overlapping the last $N - L$ samples
- Final result: $y[n]$

$y_i[n] = x_i[n] * h[n]$

Overlap-and-add is more efficient than direct when $P \geq 64$