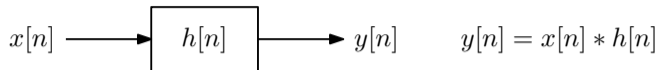


Correlation between LTI inputs and outputs

LTI correlation for system identification

Herman Kamper

Correlation between inputs and outputs of LTI systems



Cross-correlation between system output and input:

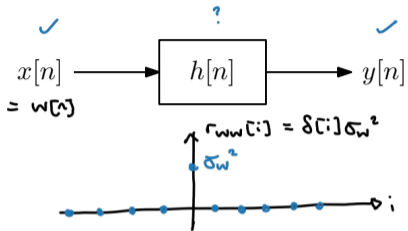
$$\begin{aligned} \Gamma_{yx}[i] &= y[i] * x[-i] \\ &= (h[i] * x[i]) * x[-i] \\ &= h[i] * (x[i] * x[-i]) \\ &= h[i] * \Gamma_{xx}[i] \end{aligned}$$

Autocorrelation:

$$\Gamma_{yy}[i] = \Gamma_{hh}[i] * \Gamma_{xx}[i]$$

Useful for knowing what system does to random signal.

Impulse response estimation

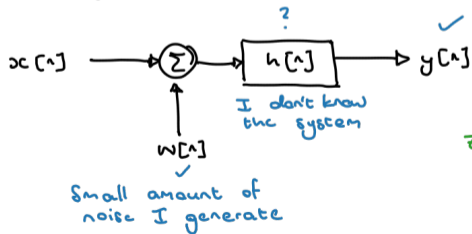


$$y[n] = x[n] * h[n]$$

$$r_{yy}[i] = h[i] * r_{xx}[i]$$

$$\begin{aligned}
 r_{yw}[i] &= h[i] * r_{ww}[i] \\
 &= h[i] * \delta[i] \sigma_w^2 \\
 &= \sigma_w^2 h[i]
 \end{aligned}$$

Online system ID:



$$\begin{aligned}
 y[n] &= (x[n] + w[n]) * h[n] \\
 &= x[n] * h[n] + w[n] * h[n]
 \end{aligned}$$

$$\begin{aligned}
 r_{yw}[i] &= y[i] * w[-i] \\
 &= (x[i] * h[i] + w[i] * h[i]) * w[-i] \\
 &= \underbrace{x[i] * h[i]}_{z[i] = x[i] * h[i]} * w[-i] + h[i] * w[i] * w[-i] \\
 &= r_{zw}[i] + h[i] * r_{ww}[i] \\
 &= \sigma_w^2 h[i]
 \end{aligned}$$