Finite impulse response (FIR) filter design

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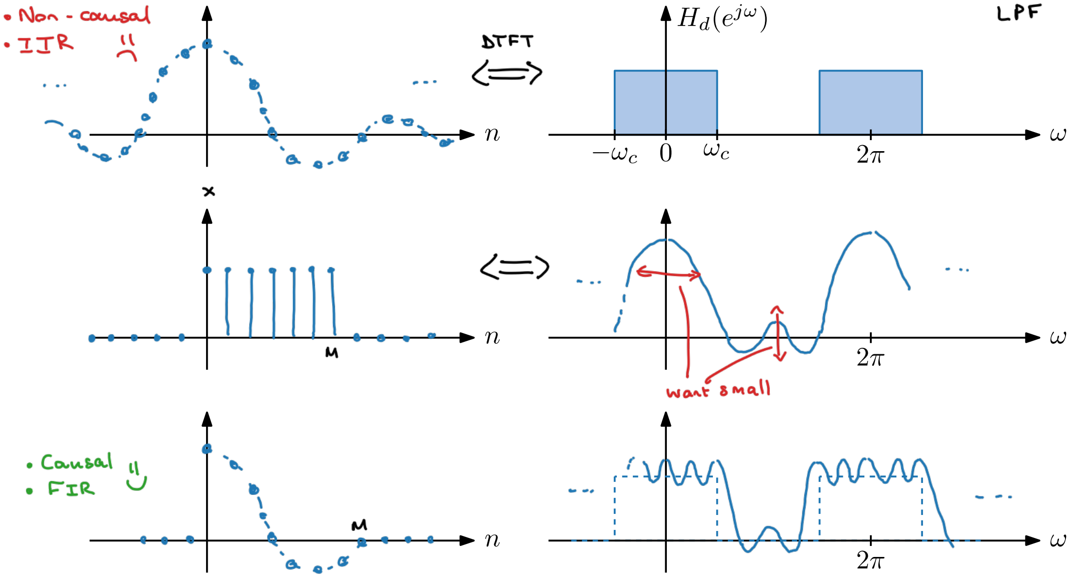
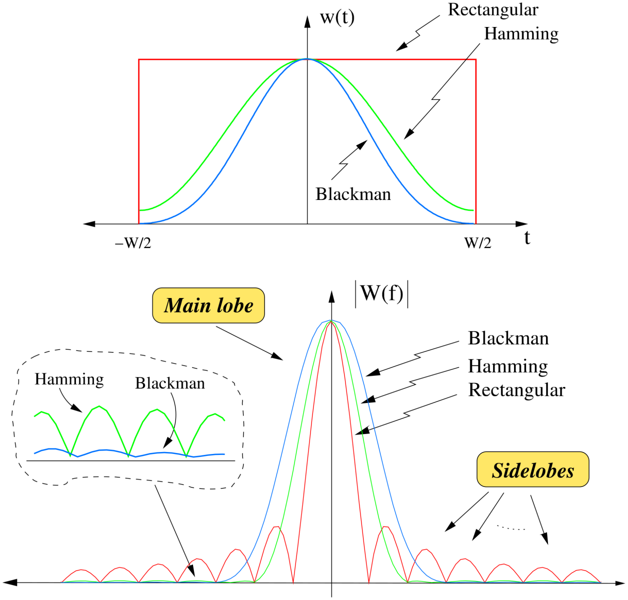
FIR filter design roadmap

1. Design ideal frequency response, get ideal impulse response
2. Throw away non-causal part and window to get practical FIR filter
3. Understand implications of this hack

FIR filter design





Other option:
Kaiser window
(has a β parameter
controlling window
shape → more control)

Linear phase in FIR filters

Linear phase: $\angle H(e^{j\omega}) = -\alpha\omega$

IDTFT

$$h[n] \overset{?}{=} \mathcal{F}^{-1}\{H(e^{j\omega})\}$$

$$= \mathcal{F}^{-1}\{\underbrace{H_R(e^{j\omega})}_{\text{Real}} \cdot e^{-j\alpha\omega}\}$$

$$= h_R[n-\alpha]$$

$H_R(e^{j\omega})$ also needs to be even, because, if $h[n]$ is real, its DTFT should have an even magnitude

$H_R(e^{j\omega})$ ← Real and even

$$h_R[n] = \mathcal{F}^{-1}\{H_R(e^{j\omega})\}$$

Real and even

(comes from DTFT properties)

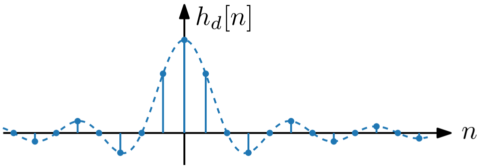
Delay: $\mathcal{F}\{x[n-k]\} = \mathcal{F}\{x[n]\} \cdot e^{-j\omega k}$

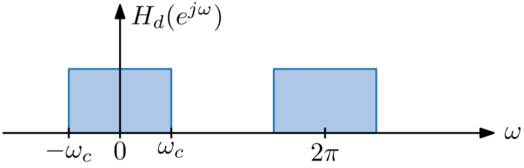
If this is true, we have linear phase :)

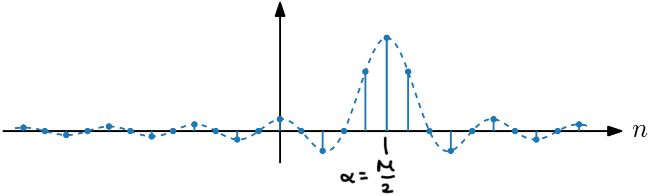
But now, in general, $h[n]$ will not be causal and will be IIR :(

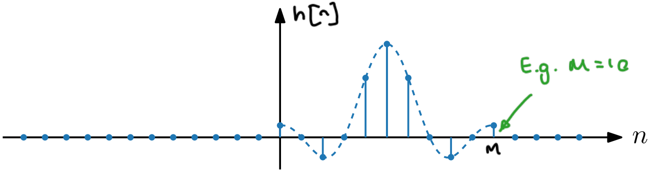
So let's make sure $h[n] = h_R[n-\alpha]$, but also causal and FIR

Linear-phase FIR filter design







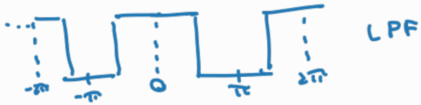


(1) Desired freq. response

(2) Take IDTFT to get $h_d[n]$
(real and even)

(3) Delay $h_d[n]$ by $\alpha = \frac{M}{2}$ samples

(4) Apply window : $h[n]$



$(e^{j\omega})\}$

$H_R(e^{j\omega})$ ← Real and even

$$h_R[n] = \mathcal{F}^{-1}\{H_R(e^{j\omega})\}$$

Real and even

$$\mathcal{F}\{x[n-k]\} = \mathcal{F}\{x[n]\} \cdot e^{-j\omega k}$$