Introduction to discrete-time systems

Characterisation and properties

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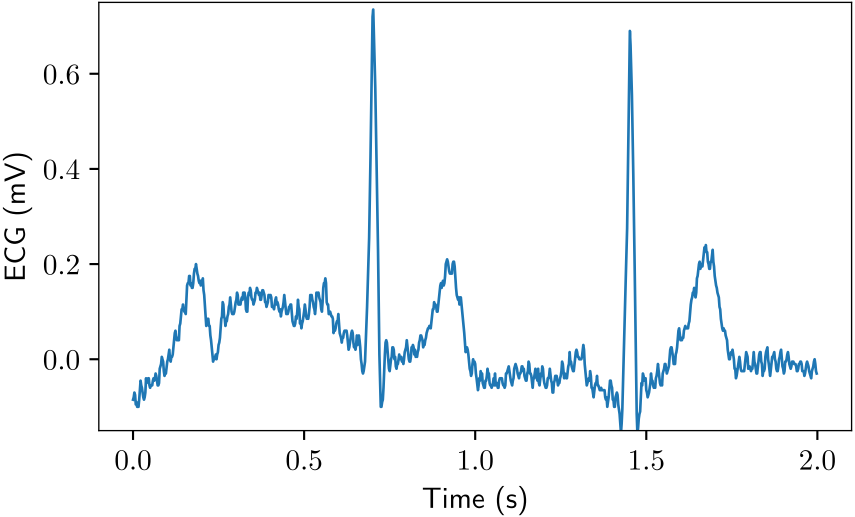
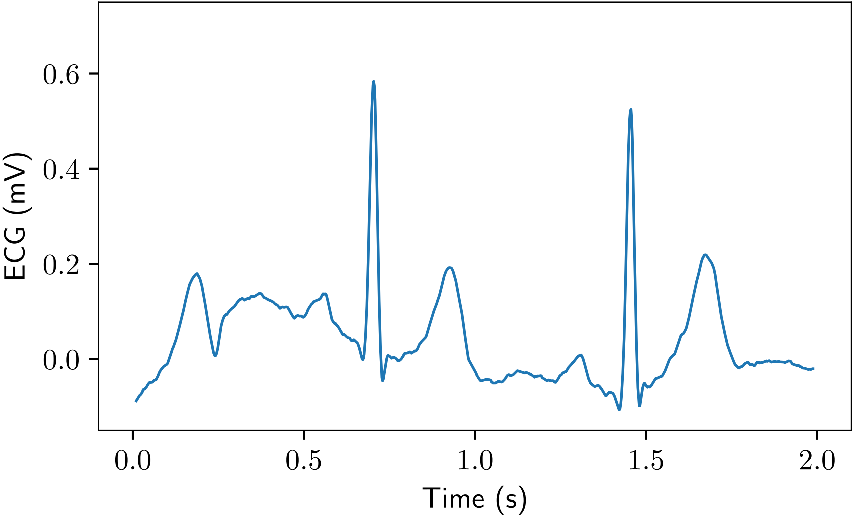

ECG (mV)
0.0
0.2
0.4
0.6
Time (s)
0.0
0.5
1.0
1.5
2.0



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0.6
0.4
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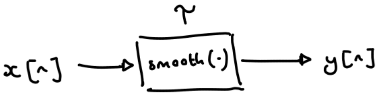
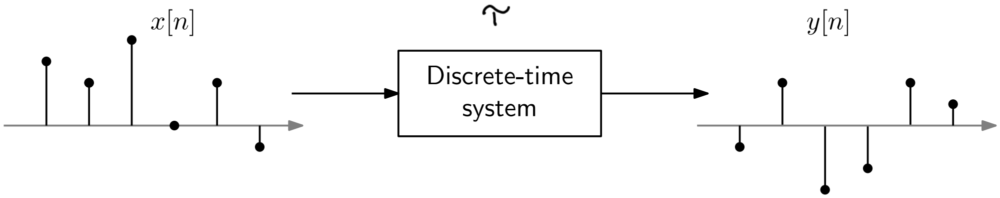
0.0
0.0
0.5
1.0
1.5
2.0
Time (s)




```
def smooth(x):
    y = []
    window = 10
    for i in range(len(x) - window):
        y.append(1 / window * np.sum(x[i : i + window]))
    return y
```



$$y[n] = \mathcal{T}\{x[n]\}$$

Called "response" of system to input $x[n]$

Example: Accumulator

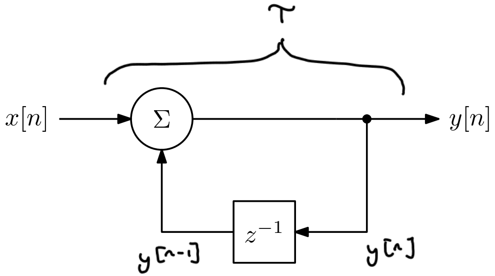
$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

Mathematical description

$$= \sum_{k=-\infty}^{n-1} x[k] + x[n]$$

$$= y[n-1] + x[n]$$

Block diagram description



Delay: z^{-1} $\quad z^{-k}$

Advance: z^{+1} $\quad z^{+k}$

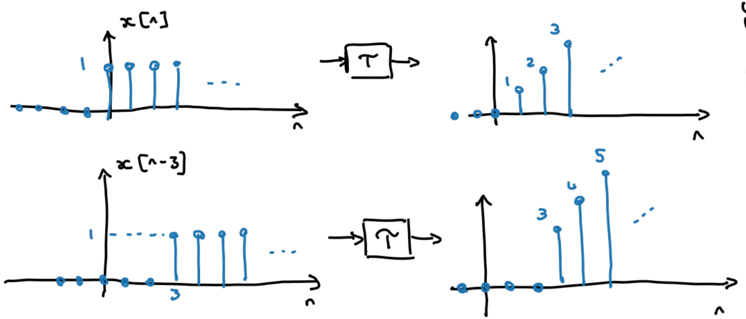
Advance by k samples

Time-invariant and time-variant systems

Time-invariant system:

$$\begin{aligned} \text{if} \quad & \mathcal{T}\{x[n]\} = y[n] \\ \text{then} \quad & \mathcal{T}\{x[n-k]\} = y[n-k] \end{aligned}$$

Example: Is $\mathcal{T}\{x[n]\} = nx[n]$ time invariant?



$y_0[n] = \mathcal{T}\{x[n]\} = n x[n]$

$y_k[n] = \mathcal{T}\{x[n-k]\} = n \cdot x[n-k]$

But $y_0[n-k] = (n-k) \cdot x[n-k]$

$\neq n \cdot x[n-k]$

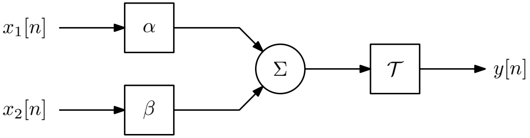
$\therefore$ system is time variant

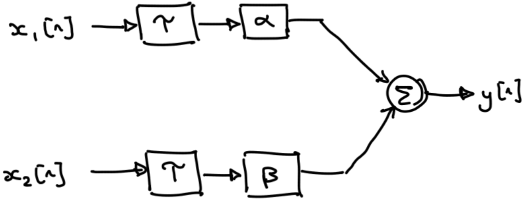
Is ChatGPT time-variant or time-invariant?

Linear systems

Linear systems obey the principle of superposition:

$$\mathcal{T}\left\{\alpha x_1[n] + \beta x_2[n]\right\} = \alpha \mathcal{T}\left\{x_1[n]\right\} + \beta \mathcal{T}\left\{x_2[n]\right\}$$





In general:

$$\boxed{\mathcal{T}\Big\{\sum_{i=1}^{N} \alpha_i x_i[n]\Big\} = \sum_{i=1}^{N} \alpha_i \mathcal{T}\left\{x_i[n]\right\}}$$

Is $\mathcal{T}\{x[n]\} = nx[n]$ a linear system?

$$\alpha \, \mathcal{T}\{x_1[n]\} = \alpha n x_1[n]$$

$$\mathcal{T}\{\alpha x_1[n] + \beta x_2[n]\}$$
$$= n(\alpha x_1[n] + \beta x_2[n])$$
$$= \alpha n x_1[n] + \beta n x_2[n]$$
$$= \alpha \, \mathcal{T}\{x_1[n]\} + \beta \, \mathcal{T}\{x_2[n]\}$$

$\therefore$ <u>Linear system</u> $\longrightarrow$

Is $\mathcal{T}\{x[n]\} = x^2[n]$ a linear system?

$$\mathcal{T}\{\alpha x_1[n] + \beta x_2[n]\}$$
$$= (\alpha x_1[n] + \beta x_2[n])^2$$
$$= \alpha^2 x_1^2[n] + 2\alpha\beta x_1[n] x_2[n] + \beta^2 x_2^2[n] \quad \cdots (1)$$

$$\alpha \, \mathcal{T}\{x_1[n]\} + \beta \, \mathcal{T}\{x_2[n]\}$$
$$= \alpha x_1^2[n] + \beta x_2^2[n] \neq (1)$$

$\therefore$ <u>Non-linear system</u> $\longrightarrow$

Causality and stability

Predictive typing: i really had a good ... in bed yesterday (GPT is causal)

Causality:

- Causal system: $y[n]$ depends only on past and present inputs, i.e. $x[k]$ for $k \leq n$
- Non-causal system: $y[n]$ depends on $x[k]$ with $k > n$

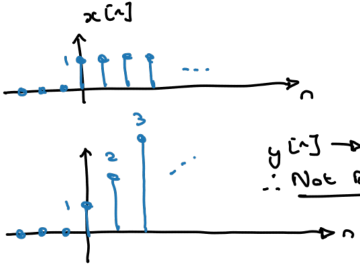
$y[n] = y[n-1] + x[n] \rightarrow$ Causal

$y[n] = x[2n] \rightarrow$ Non-causal

Bounded-input bounded-output (BIBO) stability:

if	$\lvert x[n] \rvert \leq M_x$ for all n
then	$\lvert y[n] \rvert \leq M_y$ for all n

Is the accumulator $y[n] = \sum_{k=-\infty}^{n} x[k]$ BIBO stable?



$y[n] \rightarrow \infty$

$\therefore$ Not BIBO