DFT properties and examples

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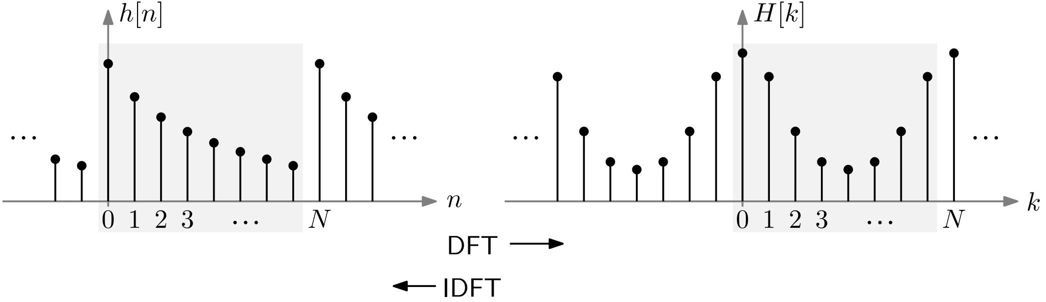

$h[n]$
n
0 1 2 3 ⋯ N
⋯
$H[k]$
k
0 1 2 3 ⋯ N
⋯
DFT
IDFT

Properties of the DFT

- Linearity:

$$\mathrm{DFT}\left\{\alpha x[n] + \beta y[n]\right\} = \alpha \mathrm{DFT}\{x[n]\} + \beta \mathrm{DFT}\{y[n]\}$$

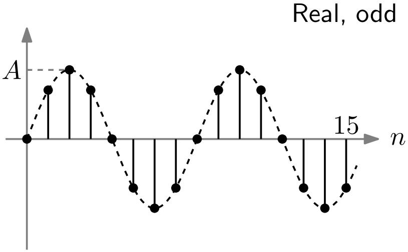
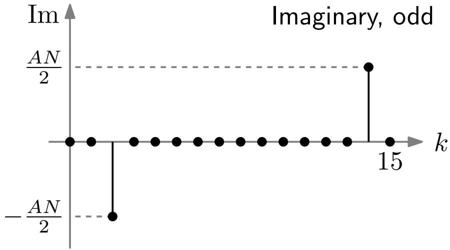
- Symmetry:

$$\text{if } \mathrm{DFT}\{h[n]\} = H[k] \text{ then } \mathrm{DFT}\{H[n]\} = N \cdot h[-k] = N \cdot h[N-k]$$

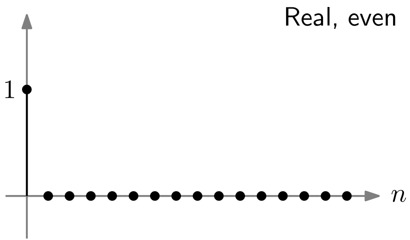
- Even and odd time sequences:
  - If $h[n]$ is even, then $h[n] = h[-n] = h[N-n]$
  - If $h[n]$ is odd, then $h[n] = -h[-n] = -h[N-n]$
  - If $h[n]$ is real, $H[k]$ has an even real and an odd imaginary part

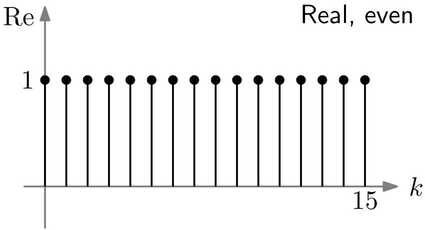
- Time reversal:

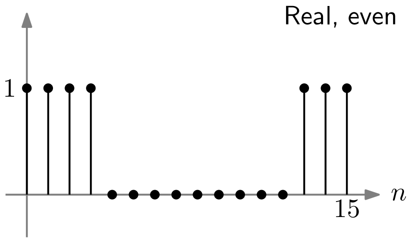
$$\mathrm{DFT}\{x[-n]\} = \mathrm{DFT}\{x[N-n]\} = X[N-k] = X[-k]$$

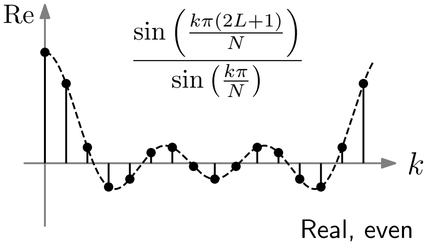
Discrete-time domain	Frequency domain
	

Discrete-time domain	Frequency domain

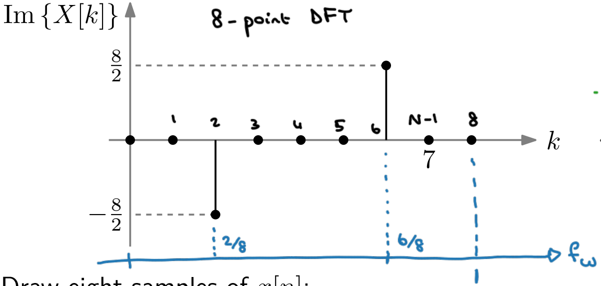


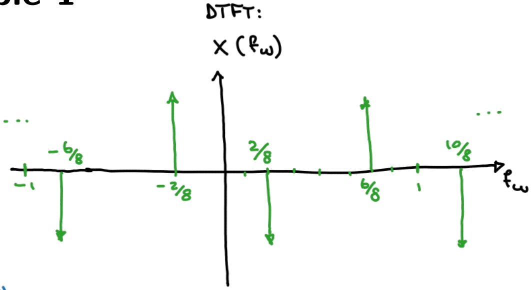






DFT example 1

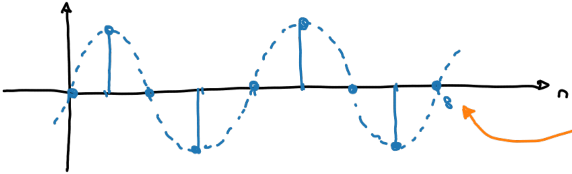




$f_{\omega,0} = \frac{2}{8} = \frac{1}{4}$ cycles/sample

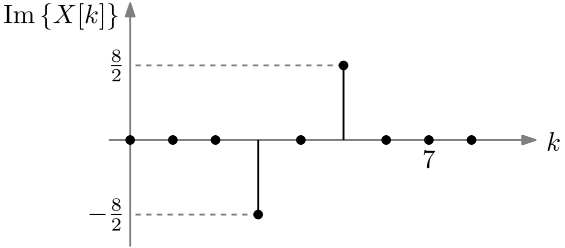
Draw eight samples of $x[n]$:

$x[n] = \sin(2\pi \cdot \frac{1}{4} n)$



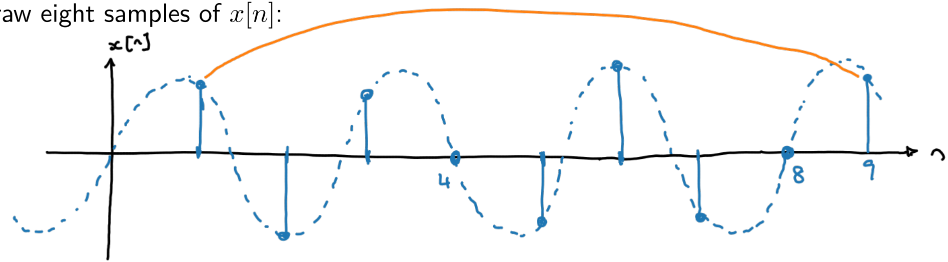
Should actually not draw, because this is the ninth sample.

DFT example 2



$f_{\omega,0} = \frac{3}{8}$ cycles/sample

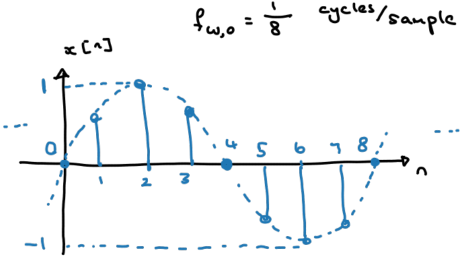
Draw eight samples of $x[n]$:

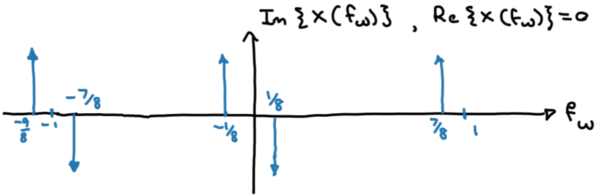


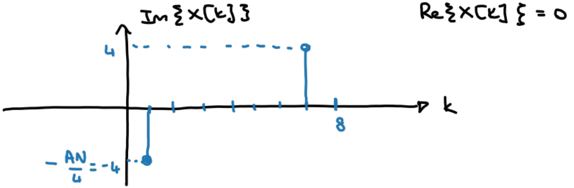
DFT example 3

$$x[n] = \sin(2\pi f_{\omega_0} n) = \sin(2\pi \frac{1}{8} n)$$

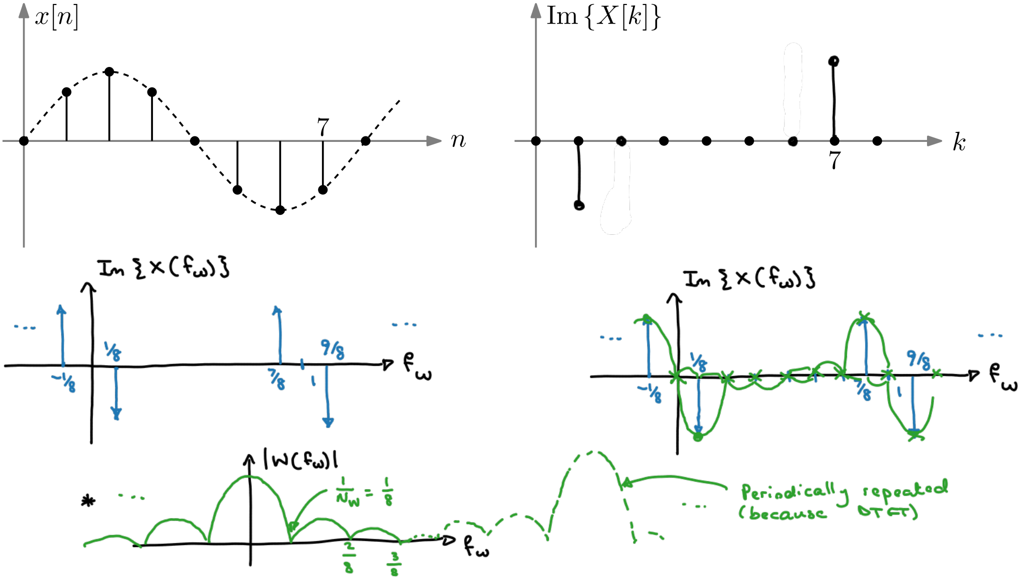
Draw $x[n]$ and its eight-point DFT $X[k]$:







Where did the side lobes go?



Further watching

B. Douglas, Understanding the discrete Fourier transform and the FFT, 2023.

Veritasium, The most important algorithm of all time, 2022.