# Learning Dynamics of Linear Denoising Autoencoders

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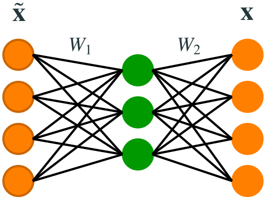
# Linear denoising autoencoders (DAE)



Expected reconstruction loss

$$\frac{1}{2N}\sum_{i=1}^{N}\mathbb{E}\left[||\mathbf{x}_i - W_2 W_1 \tilde{\mathbf{x}}_i||^2\right]$$

$$\Downarrow$$

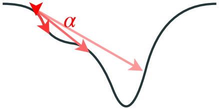
$$W_2 W_1 \approx I \qquad \Sigma_{xx} = \sum_{i=1}^{N}\mathbf{x}_i\mathbf{x}_i^T = V\Lambda V^T$$

$$\frac{\lambda}{2\tau}(1 - w_2 w_1)^2 + \frac{\varepsilon}{2\tau}(w_2 w_1)^2$$

$$\Downarrow$$

$$w(t, \lambda, \varepsilon) \approx 1$$

Continuous time analysis



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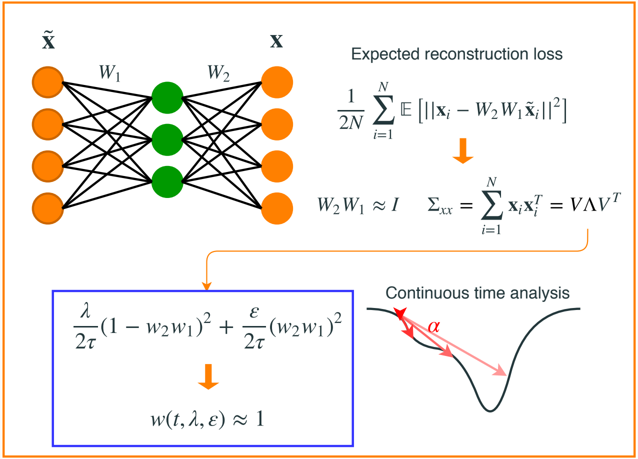

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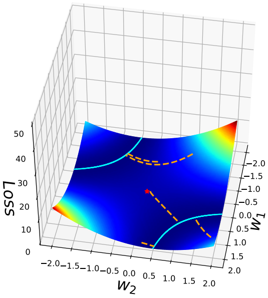
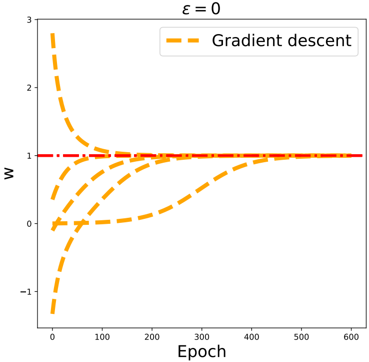
Continuous time analysis

- *Exact solutions to the nonlinear dynamics of learning in deep linear neural networks*, Saxe, McClelland, Ganguli. ICLR, 2014.

# Learning dynamics

$$\ell_{\varepsilon} = \frac{\lambda}{2\tau}(1 - w_2 w_1)^2 + \frac{\varepsilon}{2\tau}(w_2 w_1)^2 \longrightarrow \text{GD learning dynamics}$$





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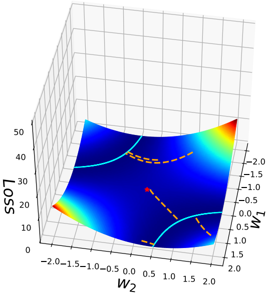

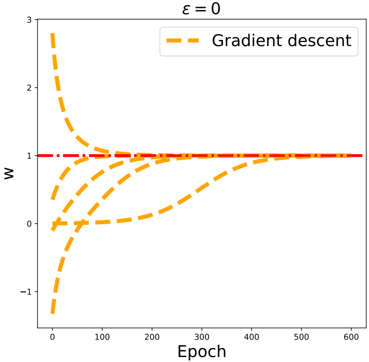

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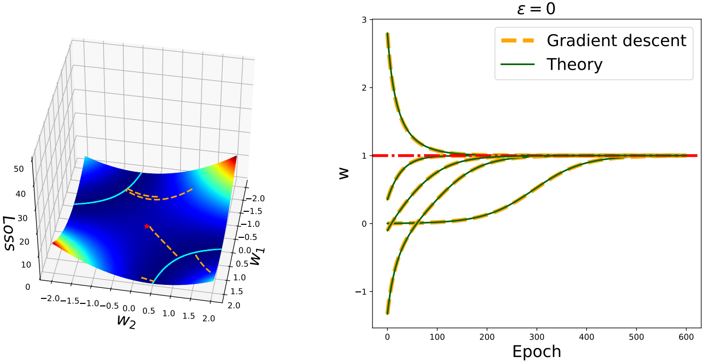

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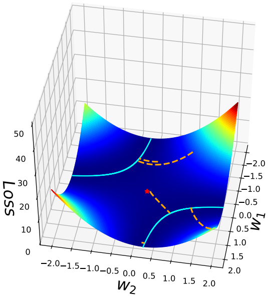

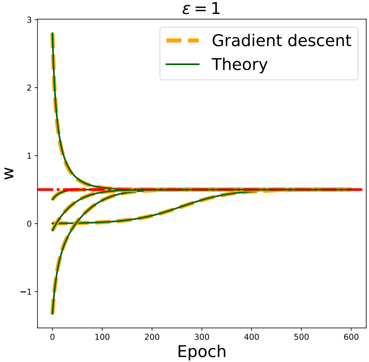

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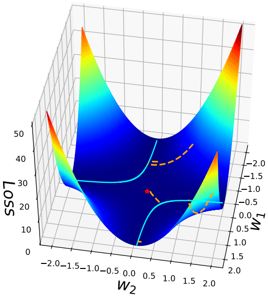

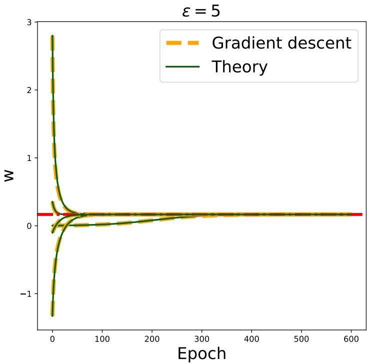

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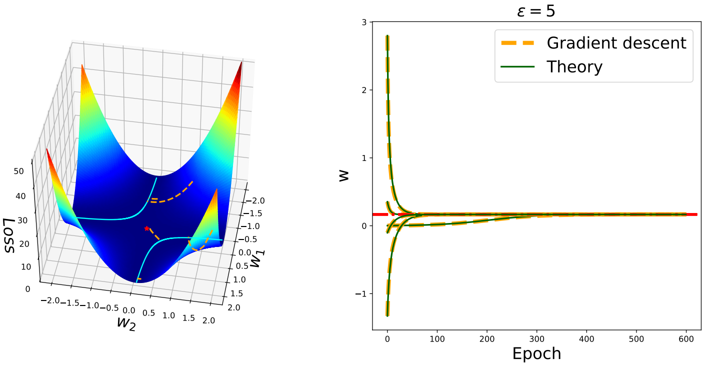
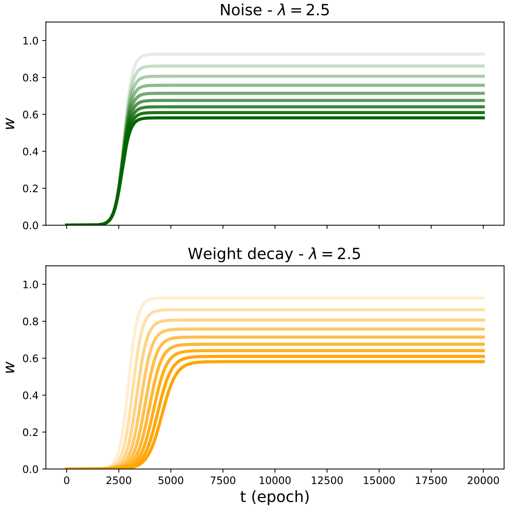


- Fixed point: $w^* = \frac{\lambda}{\lambda+\varepsilon}$

# The relationship between noise and weight decay



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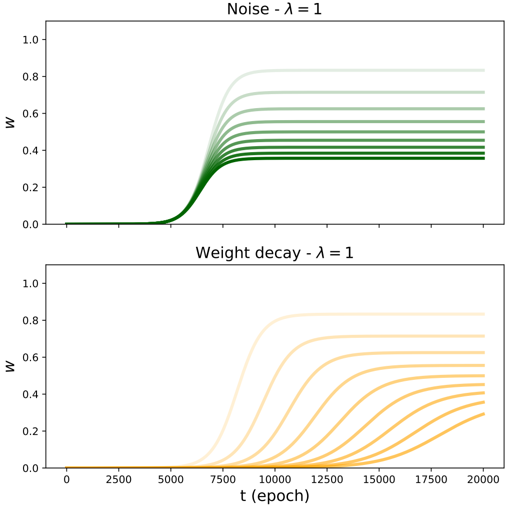

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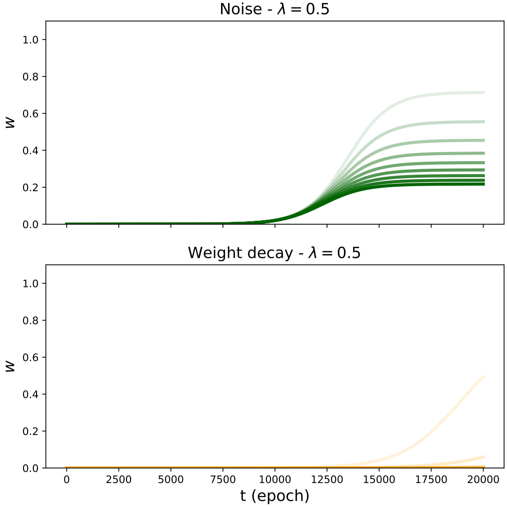
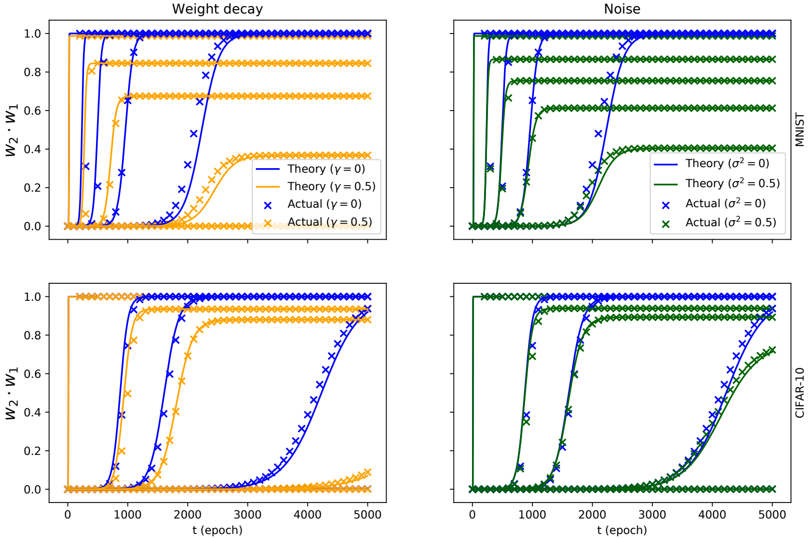
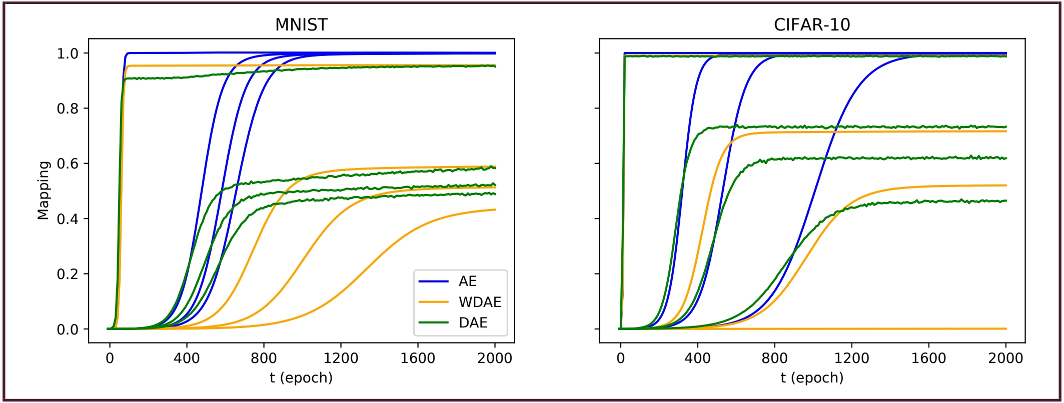


# Experimental results: Linear autoencoder networks



# Experimental results: Nonlinear autoencoders using ReLU



**Thank you for listening!**

**Source code to reproduce all the results**

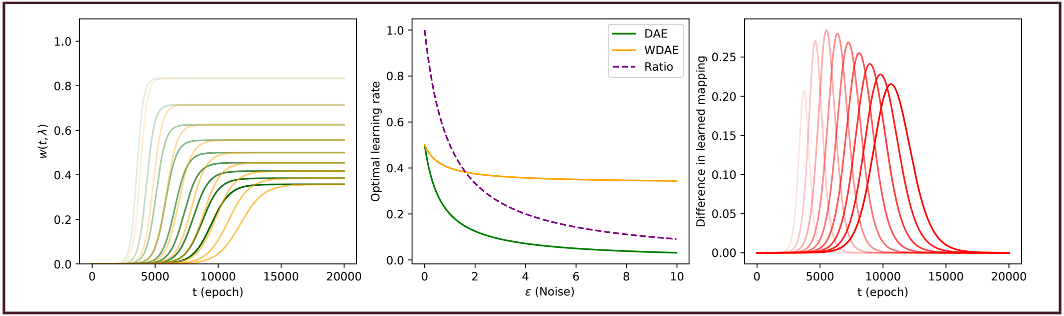
https:
//github.com/arnupretorius/lindaedynamics_icml2018

# The relationship between noise and weight decay

## Optimal discrete time learning rates

- Ratio for DAE to WDAE:

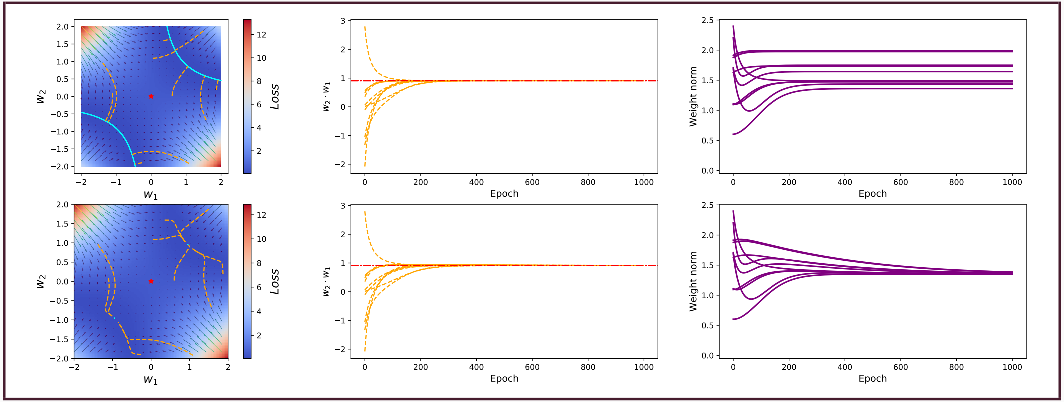
$$R = \frac{2\lambda + \gamma}{2\lambda + 3\varepsilon}.$$



# The relationship between noise and weight decay

## Motivation for weight decay

- Smaller weights $\Longrightarrow$ smoother models $\Longrightarrow$ better generalisation



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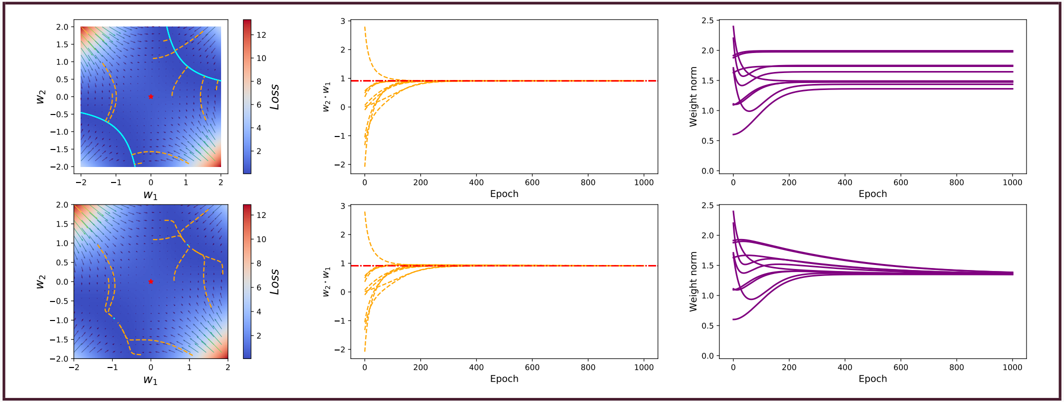
## Motivation for weight decay

- Smaller weights $\implies$ smoother models $\implies$ better generalisation
- Small weight initialisation?



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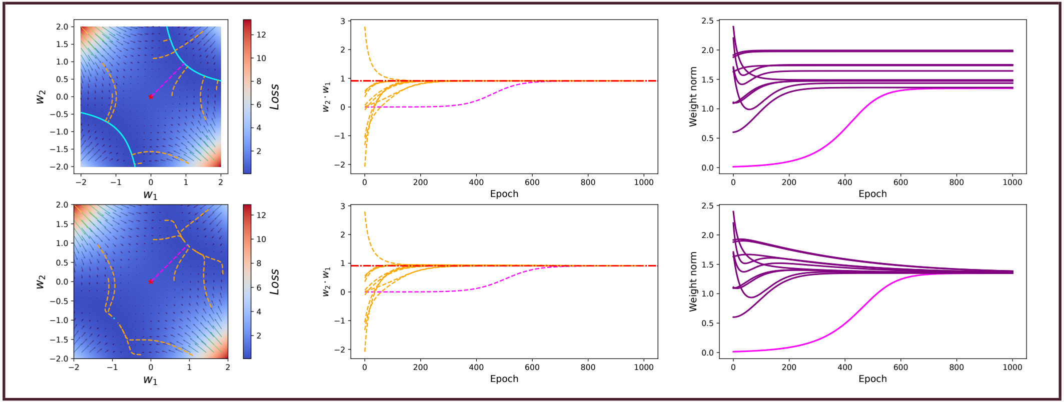
$w_1$
$w_2$
Loss
$w_2 \cdot w_1$
Epoch
Weight norm