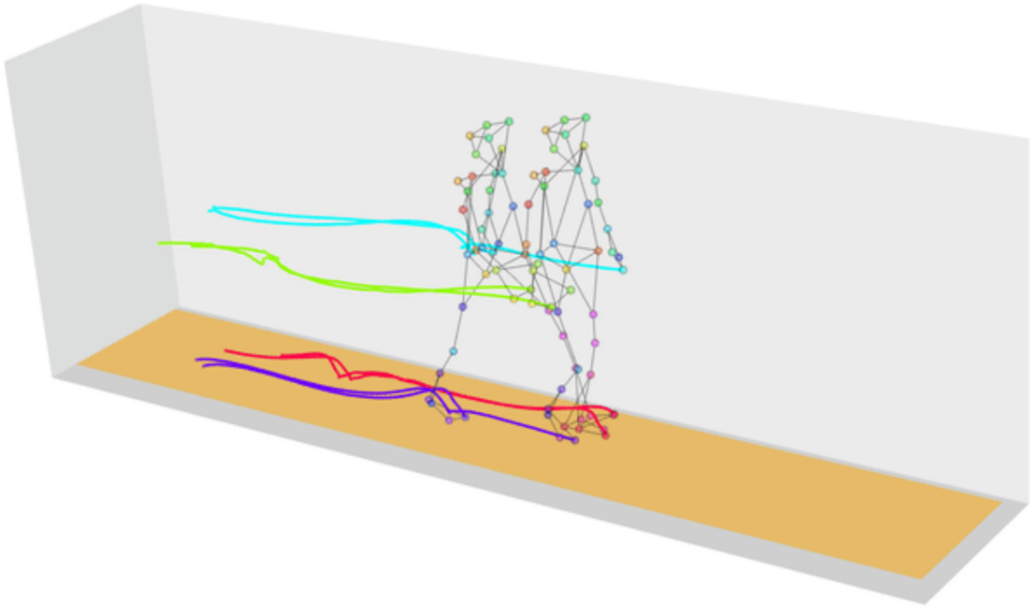
Dynamic Time Warping

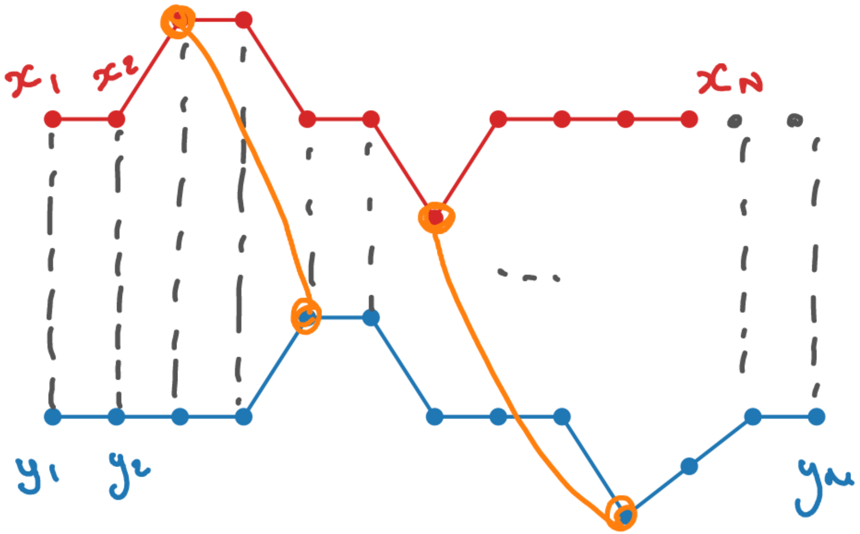
Herman Kamper

http://www.kamperh.com/

Motivation for DTW

- How similar are two signals?
- Which points correspond to one another?





Naive approach:

$$d(x_{1:N}, y_{1:M}) = \sum_{\forall i} |x_i - y_i|$$

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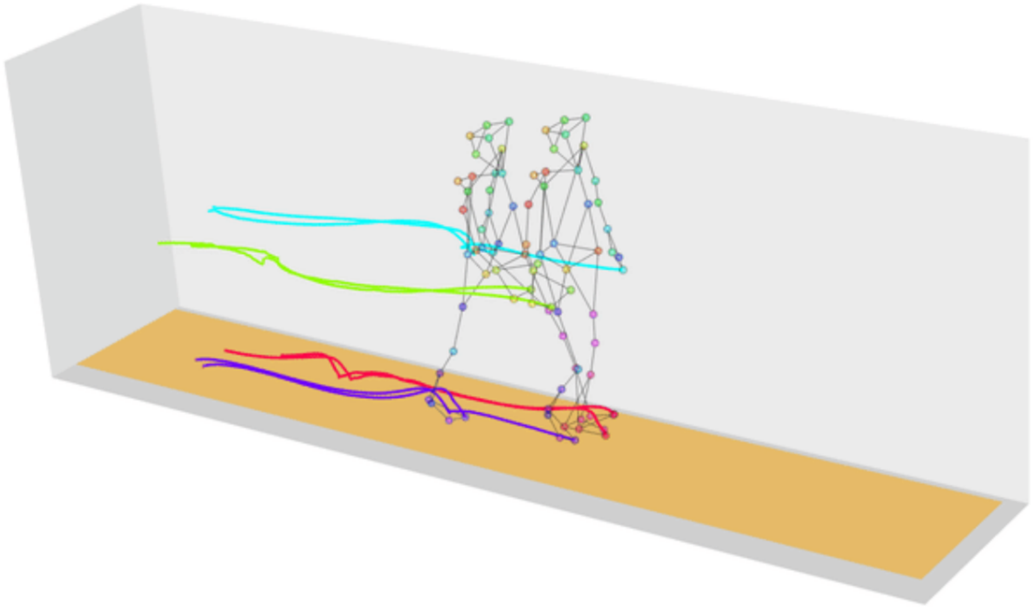
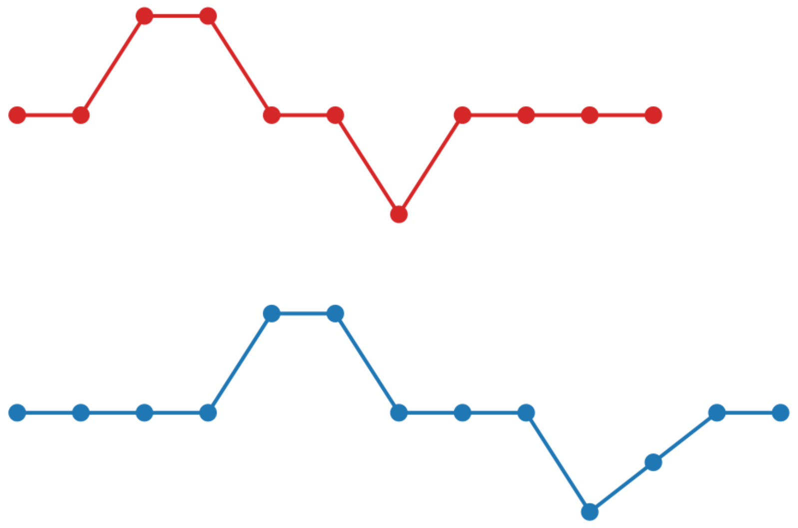
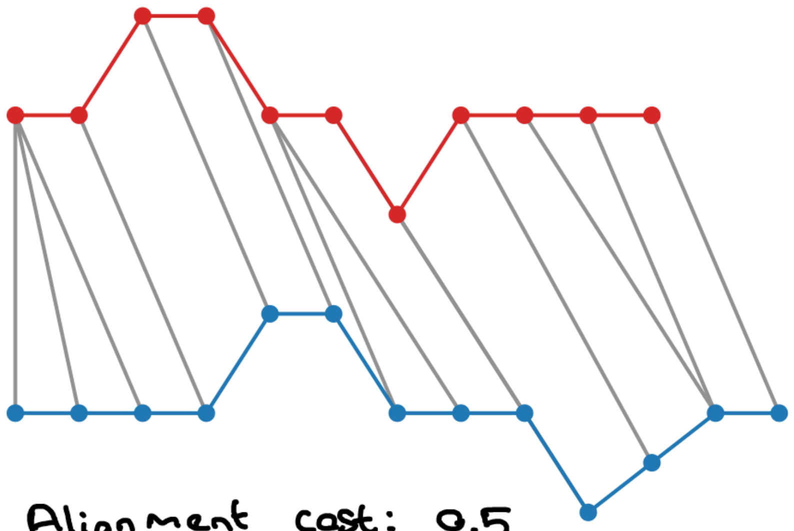
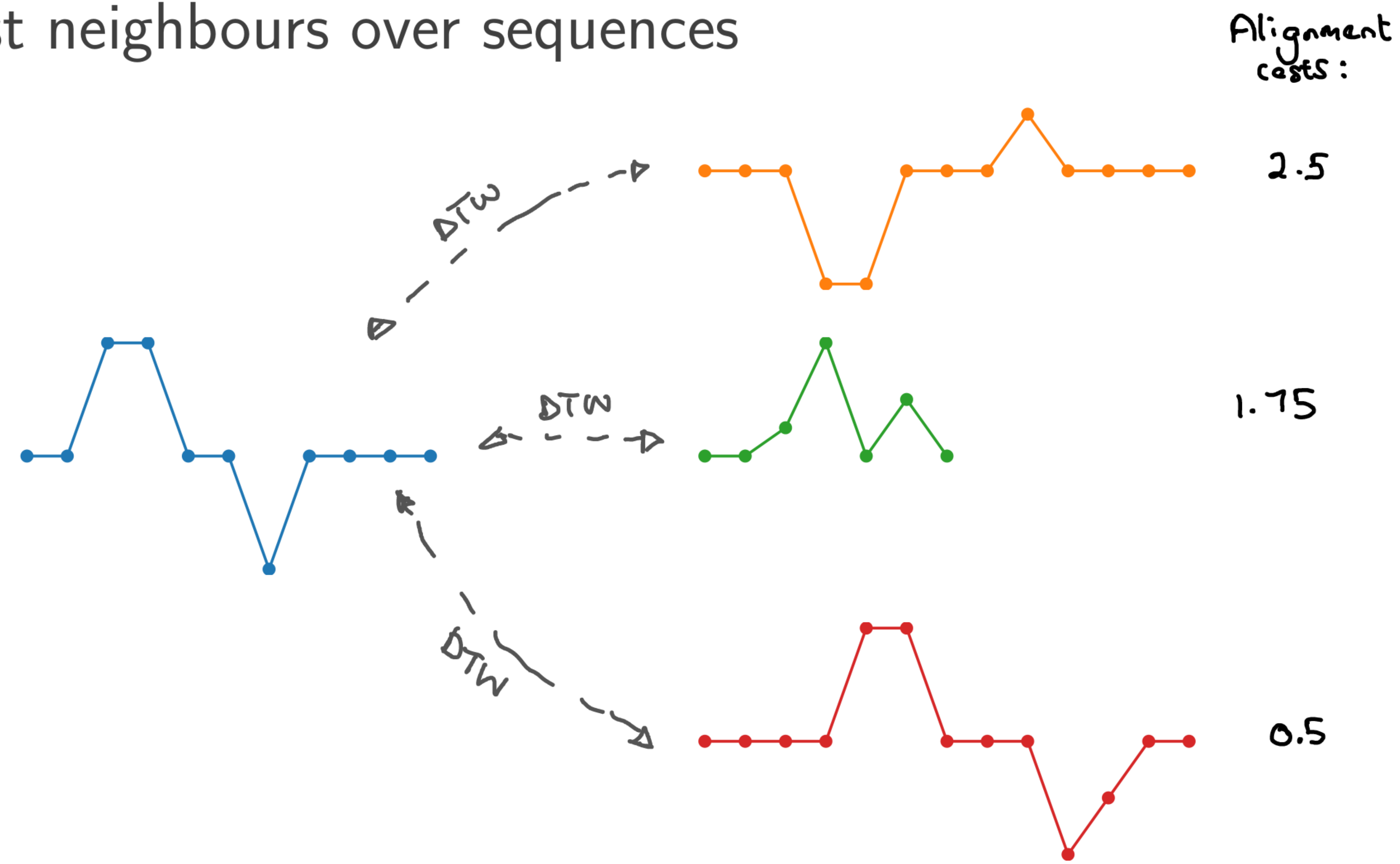






Alignment cost: 0.5

K-nearest neighbours over sequences



DTW algorithm

- Inputs: $x_{1:N}$ and $y_{1:M}$
- Cost matrix: $\mathbf{D} \in \mathbb{R}^{(N+1)\times(M+1)}$
- Initialization:

  for $i = 1$ to N: $D_{i,0} = \infty$

  for $j = 1$ to M: $D_{0,j} = \infty$

  $D_{0,0} = 0$
- Calculate cost matrix:

  for $i = 1$ to N:

  for $j = 1$ to M:

$$D_{i,j} = d(x_i, y_j) + \min \begin{cases} D_{i-1,j-1} & \text{(match)} \\ D_{i-1,j} & \text{(insertion)} \\ D_{i,j-1} & \text{(deletion)} \end{cases}$$

- Get alignment: Trace back from $D_{N,M}$ to $D_{0,0}$

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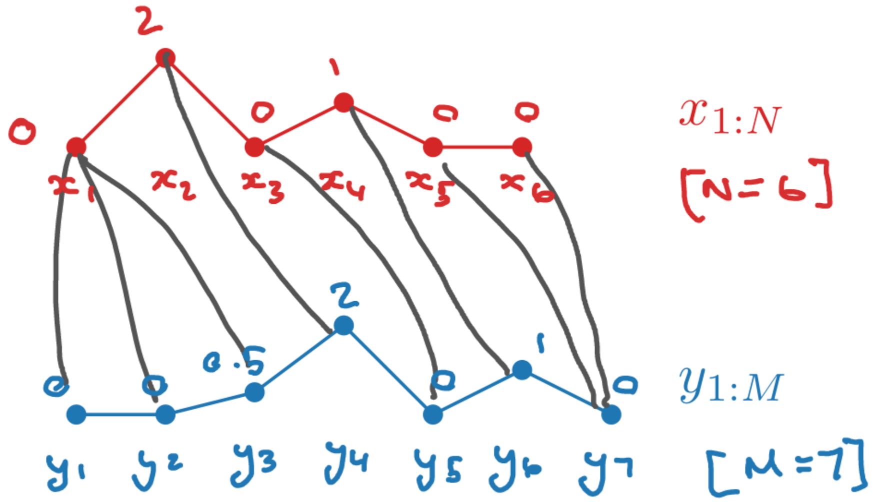

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$d(x_i, y_j) = |x_i - y_j|$

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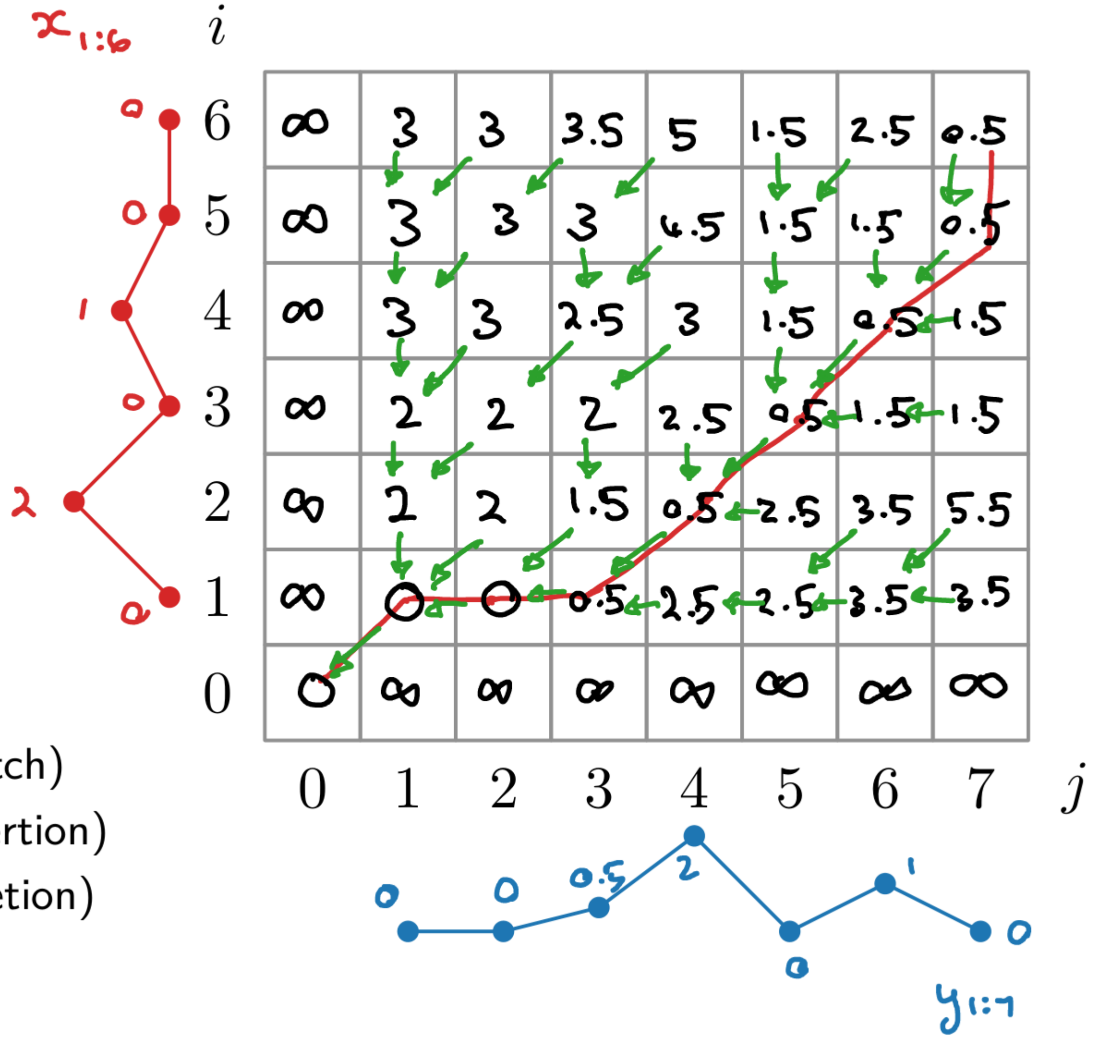
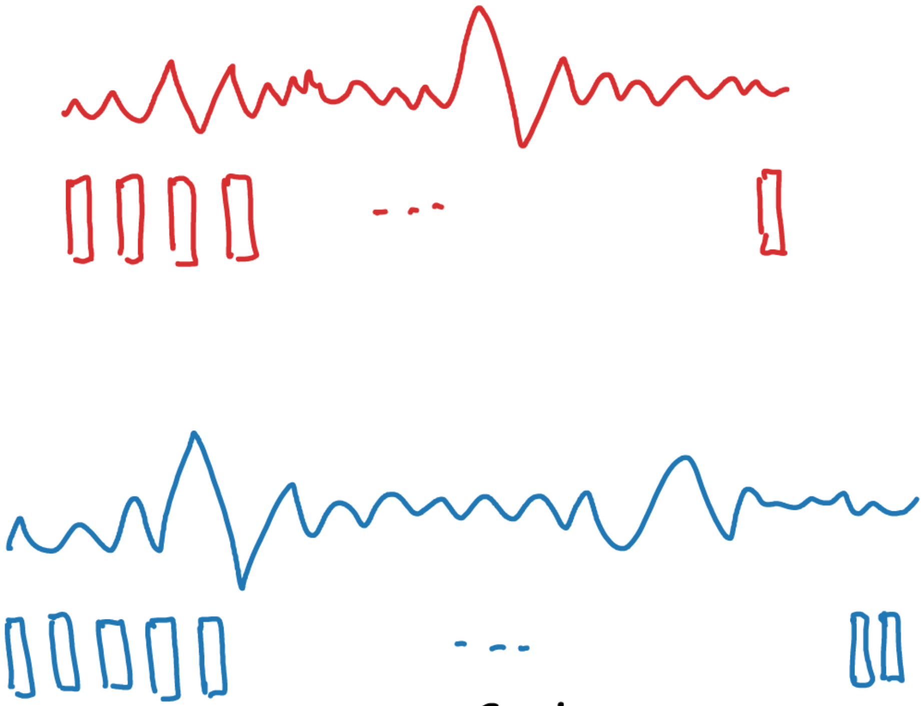


DTW on sequences of vectors



$d(\underline{x}_i, \underline{y}_j)$
- Cosine
- Dot
- Euclidean

← Options for frame-wise metric

Normalization: $\dfrac{D_{N,M}}{N+M}$

References

- D. Jurafsky, “Computing minimum edit distance,” Stanford Lecture.
- D. Ellis, “Dynamic time warp (DTW) in Matlab,” http://www.ee.columbia.edu/~dpwe/resources/matlab/dtw/.
- Wikipedia, “Dynamic time warping,” https://en.wikipedia.org/wiki/Dynamic_time_warping.