

# *K*-means clustering

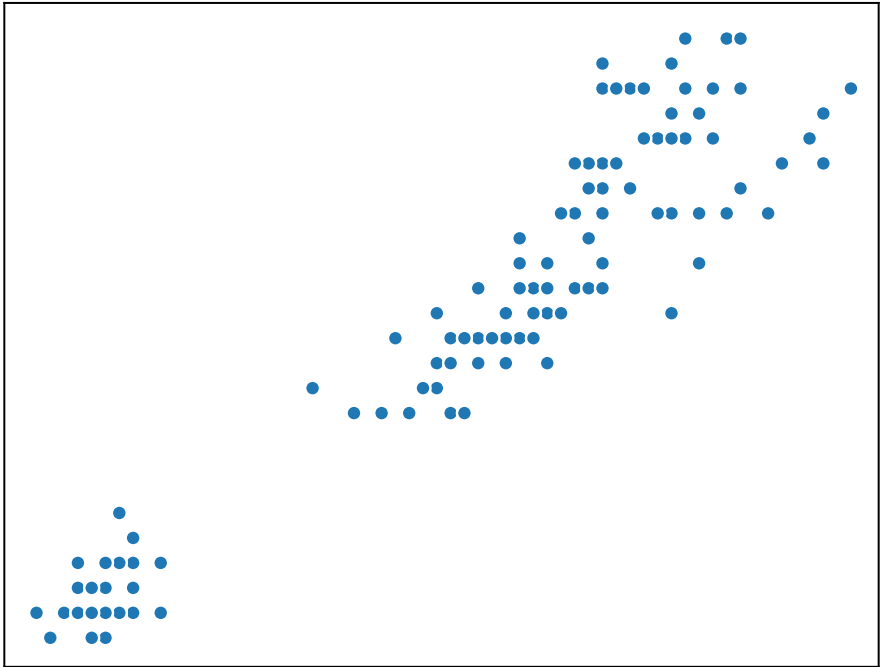
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## **$K$ -means clustering algorithm**

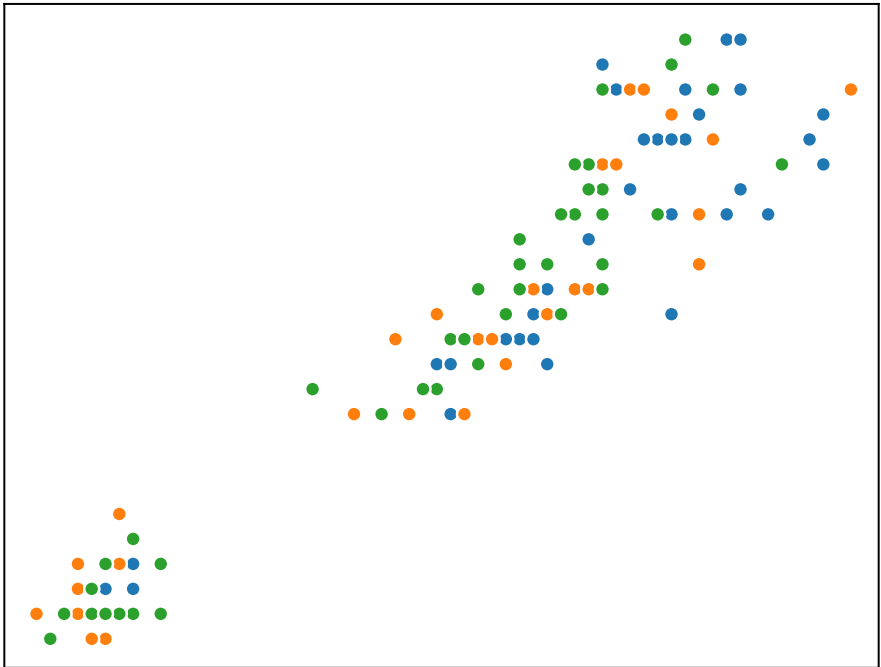
- Randomly assign each item  $\mathbf{x}^{(n)}$  to one of the  $K$  clusters.
- repeat until cluster assignments stop changing:
  - (a) for cluster  $k = 1$  to  $K$ :  
Calculate the cluster centroid  $\boldsymbol{\mu}_k$  as the mean of all the items assigned to cluster  $k$ .
  - (b) for item  $n = 1$  to  $N$ :  
Assign item  $\mathbf{x}^{(n)}$  to the cluster with the closest centroid.

# $K$ -means example



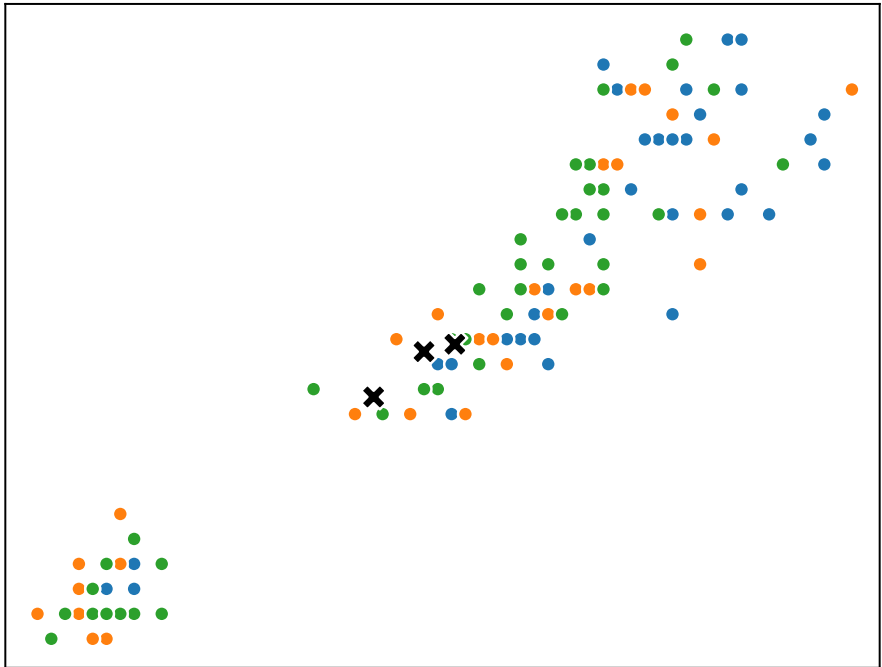
# $K$ -means example

Initialisation



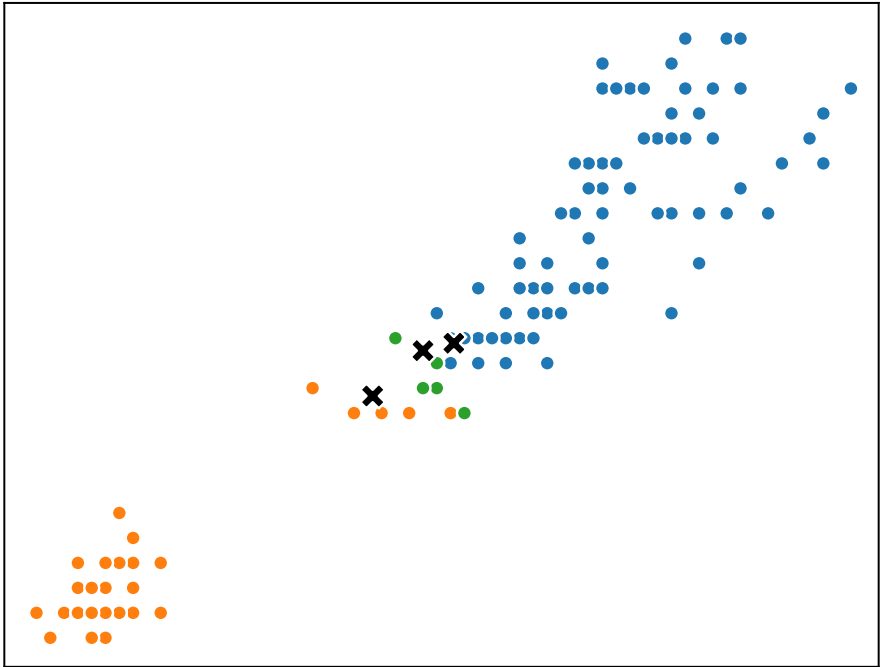
# $K$ -means example

Iteration: 1 (centroid update)



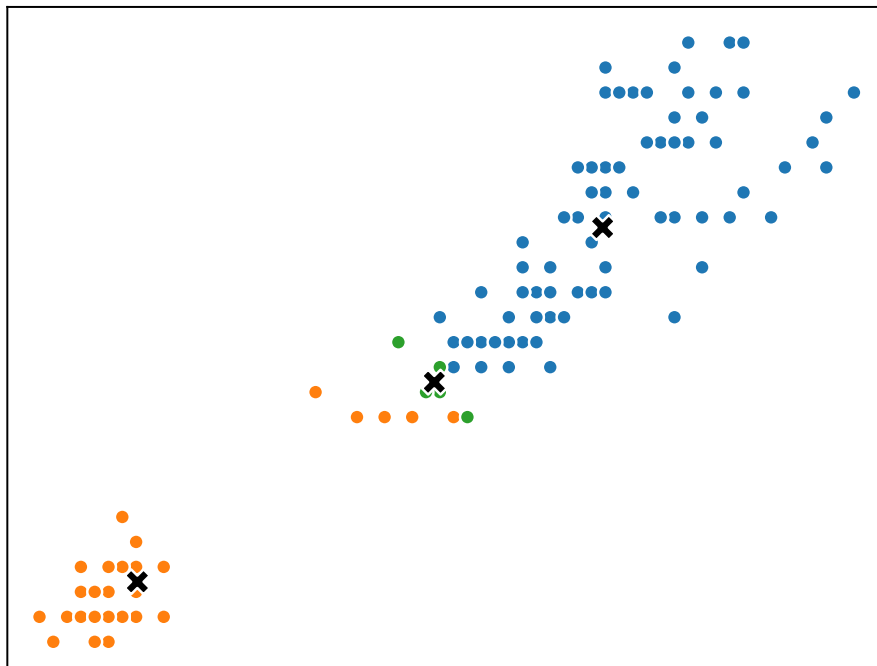
# $K$ -means example

Iteration: 1 (item assignment)



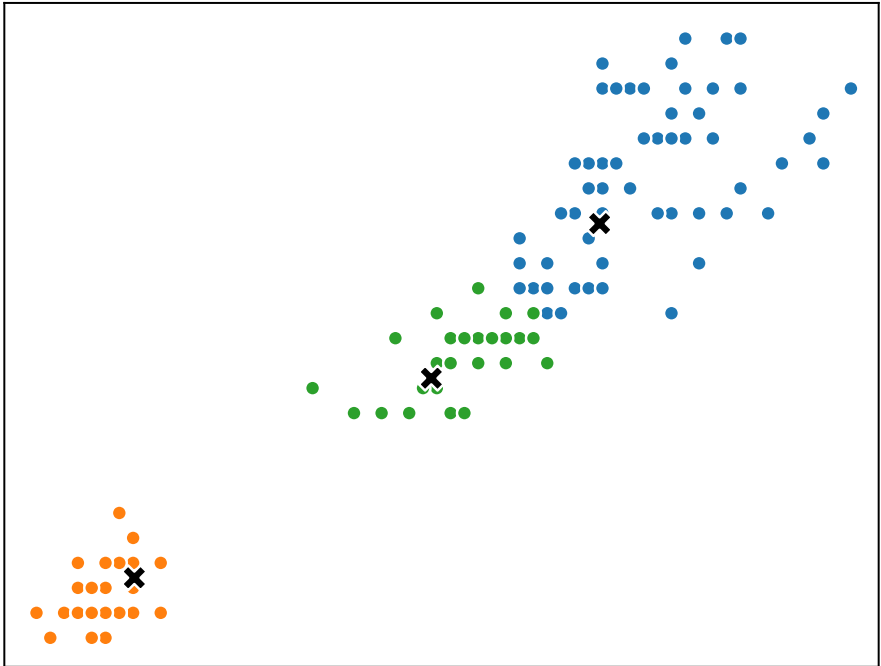
# $K$ -means example

Iteration: 2 (centroid update)



# $K$ -means example

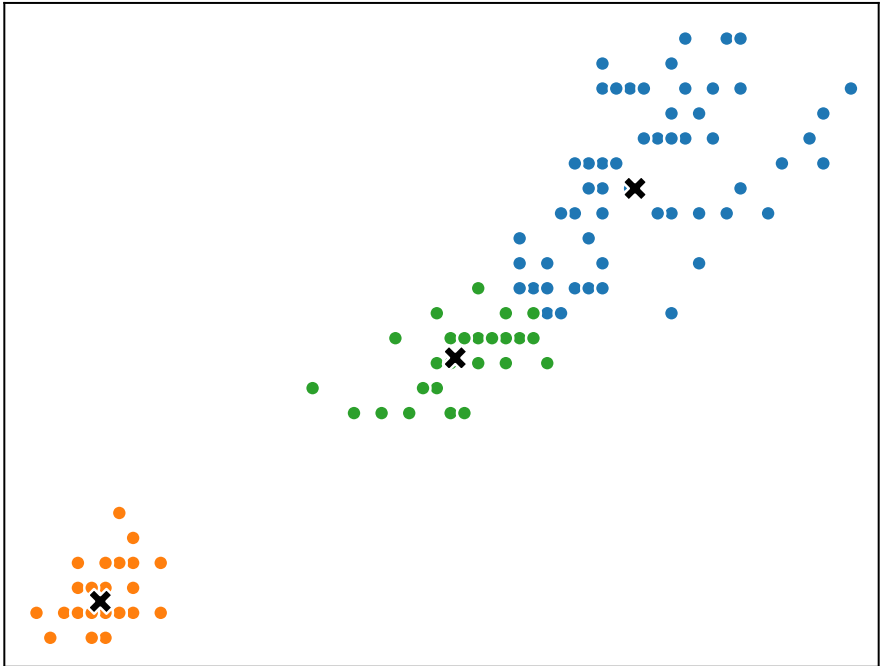
Iteration: 2 (item assignment)





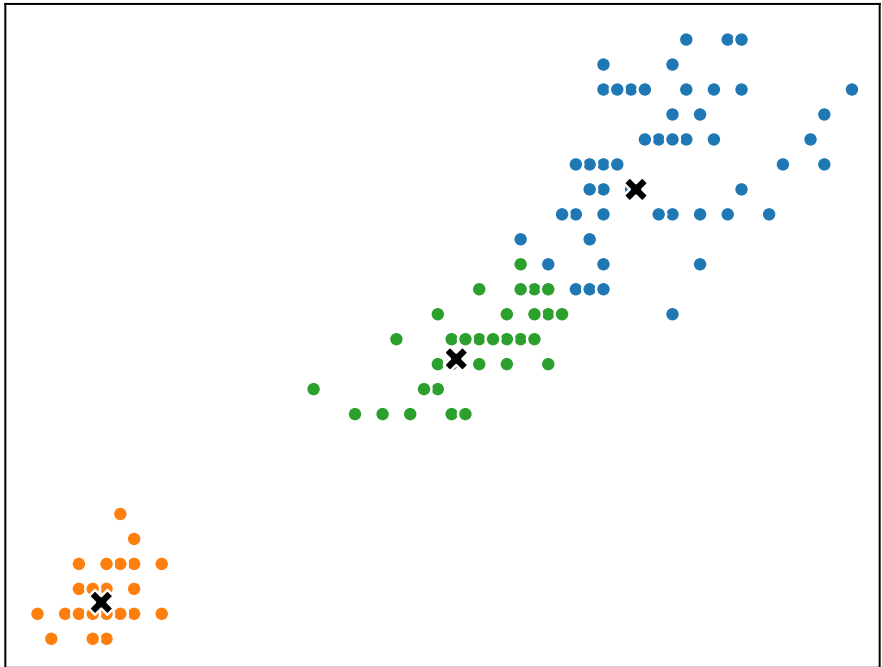
# $K$ -means example

Iteration: 3 (centroid update)



# $K$ -means example

Iteration: 3 (item assignment)



# $K$ -means clustering algorithm details

## Notation

$C_k$  denotes the set of indices of items assigned to cluster  $k$ .

$|C_k|$  denotes the number of items in cluster  $k$ .

Example:  $C_4 = \{205, 12, 303\}$ ,  $|C_4| = 3$

## Inner loop

(a) Centroid update:

Update the centroids  $\boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \dots, \boldsymbol{\mu}_K$  while keeping the cluster assignments  $C_1, C_2, \dots, C_K$  fixed.

$$\boldsymbol{\mu}_k = \frac{1}{|C_k|} \sum_{i \in C_k} \mathbf{x}^{(i)}$$

(b) Cluster assignment update:

Update the cluster assignments  $C_1, C_2, \dots, C_K$  while keeping the centroids  $\boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \dots, \boldsymbol{\mu}_K$  fixed.

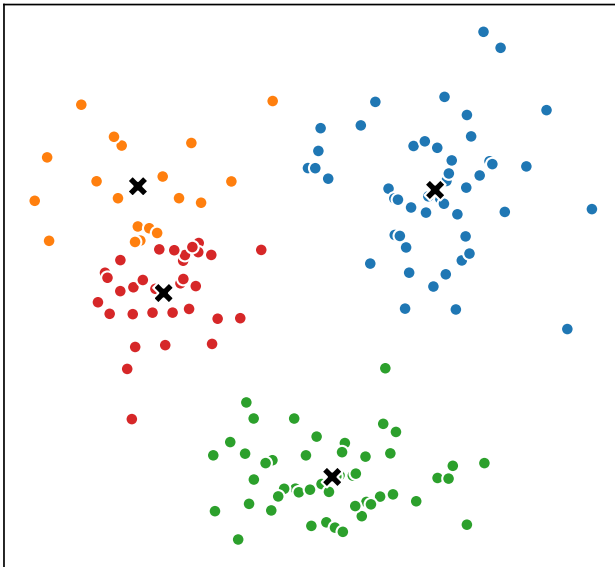
$$\arg \min_k \left\| \mathbf{x}^{(n)} - \boldsymbol{\mu}_k \right\|^2$$

## Loss

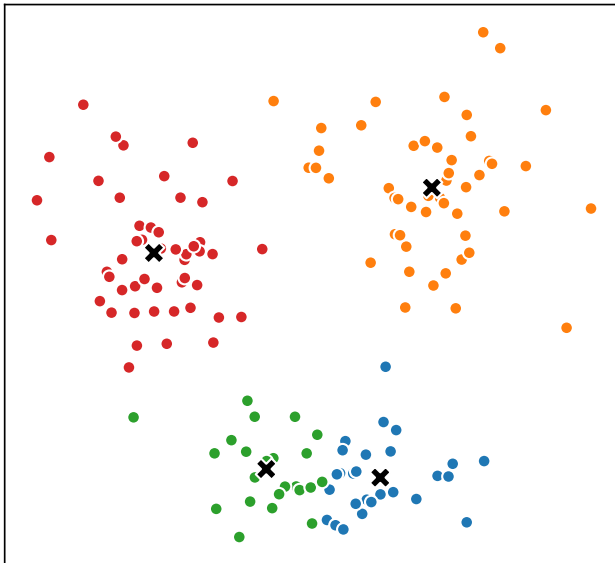
$$J(C_1, C_2, \dots, C_K, \boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \dots, \boldsymbol{\mu}_K) = \sum_{k=1}^K \sum_{i \in C_k} \left\| \mathbf{x}^{(i)} - \boldsymbol{\mu}_k \right\|^2$$

# Effect of random initialisation

Sum of squared distances to centroids: 68.26



Sum of squared distances to centroids: 66.97



## Videos covered in this note

- [K-means clustering 1 - Algorithm](#) (16 min)
- [K-means clustering 2 - Details](#) (14 min)

## Reading

- ISLR 12.4.1
- ISLR 12.4.3 - Only the content regarding  $K$ -means clustering is examinable (not hierarchical clustering).